OPERATIONS RESEARCH

Time allowed: 3 hrs **Max. Marks: 100**

(Answer any five questions)

Each question carries 20 marks **(5 x 20 = 100 marks)**

1. Mention the principals of modeling and briefly explain the different models used in Operations Management.

2. A finished product must exactly weigh 150 grams. The two raw materials used in manufacturing the product are RI, with a cost of Rs 2 per unit and R2 with a cost of Rs 8 per unit. At least 14 units of R_2 and not more than 20 units of R_1 must be used. Each unit of R_1 and R_2 weighs 5 and 10 grams respectively. How much of each type of raw material should be used for each unit of the final product if cost is to be minimised? Formulate the above situation as a mathematical model.

3.Find the initial solution of the following Transportation problem:

		To			
		A	B	C	Available
	I	50	30	220	1
From	II	90	45	170	3
	III	250	200	50	4
	Requirement	4	2	2	

4. Each year Blue Cross Hospital purchases 20000 syringes that cost Rs.16 per syringe. The cost of placing an order is Rs.12 and the cost of holding is 24% per year.
   i. Determine the economic order quantity.
   ii. Compute the average inventory level, assuming that minimum inventory level is zero.
   iii. Estimate the number of orders per year and time between orders.
   iv. Determine the total annual cost.

5. A maintenance activity in the hospital consists of following jobs. Draw the network for the project and calculate the total float and free float for each activity. What can you say about the slacks of the events of the project?

Job	Duration (in days)
1-2	2
1-3	4
1-4	3
2-5	1
3-5	6
4-6	5
5-6	7

6. A department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix. How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

	Employees				
	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	1	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

7. Following table indicates the details of a project. The durations are in days :

Activity	t_0	t_m	t_p
1-2	2	4	5
1-3	3	4	6
1-4	4	5	6
2-4	8	9	11
2-5	6	8	12
3-4	2	3	4
4-5	2	5	7

i. Draw the network

ii. Find the critical path

iii. Determine the expected variance of the completion time.

8. Write short notes on ***any four*** :

i. Dummy Activity.
ii. Assignment problem in hospital operations management.
iii. Fast tracking and Crashing.
iv. Statistical Control charts in hospital operations.
v. Queuing and capacity planning in Hospitals.

Solution

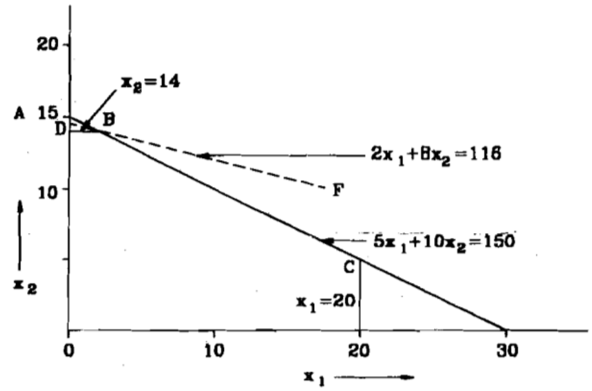
Answer 2

We introduce decision variables x_1 and x_2 indicating the number of units of raw material A and raw material B respectively. Then the problem can be formulated as

Minimise $2x_1 + 8x_2$

Subject to :

$$5x_1 + 10x_2 \geq 150$$
$$x_1 \leq 20$$
$$x_2 \geq 14$$
$$x_1, x_2 \geq 0$$



The feasible region of the problem is the triangle ADB. The line $2x_1 + 8x_2 = 116$ passes through the extreme point B. Hence, the optimum solution of the problem is given by $x_1 = 2$, $x_2 = 14$ with a minimum cost of Rs. 116.

Answer 3

Total number of supply (available) constraints : 3
Total number of demand (requirement) constraints : 3

Problem Table is

	A	*B*	*C*	Available
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Requirement	4	2	2	

The rim values for *I*=1 and *A*=4 are compared.

The smaller of the two i.e. $\min(1,4) = 1$ is assigned to *I A*

This exhausts the capacity of *I* and leaves 4 - 1 = 3 units with *A*

Table-1

	A	B	C	Available
I	~~50(1)~~	~~30~~	~~220~~	0 0=1-1
II	90	45	170	3
III	250	200	50	4
Requirement	3 3=4-1	2	2	

The rim values for II=3 and A=3 are compared.

The smaller of the two i.e. $\min(3,3)$ = **3** is assigned to II A

This exhausts the capacity of II and leaves 3 - 3 = 0 units with A

Table-2

	A	B	C	Available
I	~~50(1)~~	~~30~~	~~220~~	0
II	~~90(3)~~	~~45~~	~~170~~	0 0=3-3
III	250	200	50	4
Requirement	0 0=3-3	2	2	

The rim values for III=4 and A=0 are compared.

The smaller of the two i.e. $\min(4,0)$ = **0** is assigned to III A

This meets the complete demand of A and leaves 4 - 0 = 4 units with III

Table-3

	A	B	C	Available
I	~~50(1)~~	~~30~~	~~220~~	0
II	~~90(3)~~	~~45~~	~~170~~	0
III	250	200	50	4 4=4-0
Requirement	0 0=0-0	2	2	

The rim values for III=4 and B=2 are compared.

The smaller of the two i.e. $\min(4,2)$ = **2** is assigned to III B

This meets the complete demand of B and leaves 4 - 2 = 2 units with III

Table-4

	A	*B*	*C*	Available
I	~~50(1)~~	~~30~~	~~220~~	0
II	~~90(3)~~	~~45~~	~~170~~	0
III	250	200**(2)**	50	2 2=4-2
Requirement	0	0 0=2-2	2	

The rim values for *III*=2 and *C*=2 are compared.

The smaller of the two i.e. min(2,2) = **2** is assigned to *III C*

Table-5

	A	*B*	*C*	Available
I	~~50(1)~~	~~30~~	~~220~~	0
II	~~90(3)~~	~~45~~	~~170~~	0
III	~~250~~	~~200(2)~~	~~50(2)~~	0 0=2-2
Requirement	0	0	0 0=2-2	

Initial feasible solution is

	A	*B*	*C*	Available
I	50 **(1)**	30	220	1
II	90 **(3)**	45	170	3
III	250	200 **(2)**	50 **(2)**	4
Requirement	4	2	2	

The minimum total transportation cost $=50\times1+90\times3+200\times2+50\times2=820$

Here, the number of allocated cells = 4, which is one less than to m + n - 1 = 3 + 3 - 1 = 5
∴ This solution is degenerate

Answer 4

Annual Purchase (A) = 20000 syringes
Cost per syringe (C) = Rs 16/-
Cost per order (S) = Rs 12/-
Holding cost or Inventory carrying cost (I@24%) = 0.24
EOQ = √((2AS)/(CI))

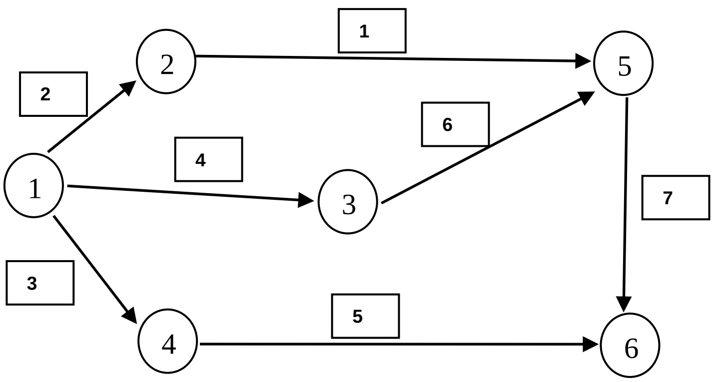
$$EOQ = \sqrt{\frac{2*20000*12}{16*0.24}}$$

EOQ = 353.55 (say 354 units)

Number of order per year = 20000/354 = 56.49 (say 56 orders)
Time between two orders = 56/12 = 4.7 months

Answer 5

Network Diagram



Critical Path

Path	**Duration of Path (in days)**
1-2-5-6	2+1+7 = 10
1-3-5-6	4+6+7 = 17
1-4-6	3+5 = 8

Hence, the critical path is 1-3-5-6 with duration of 17 days

Computation of Earliest Start Time (ES)

Node	Activities leading to Node	Sum of expected time	ES (being the maximum of expected time)
1	-	0	0
2	1-2	2	2
3	1-3	4	4
4	1-4	3	3
5	1-2 , 2-5 1-3 , 3-5	2+1 = 3 4+6 = 10	10
6	1-2 , 2-5 , 5-6 1-3 , 3-5 , 5-6 1-4 , 4-6	2+1+7 = 10 4+6+7 = 17 3+5 = 8	17

Computation of Latest Finish Time (LF)

Node (Column 1)	Activities leading back to Node (Column 2)	Sum of expected time (Column 3)	Maximum of sum of expected time (Column 4)	LF = 17 – Column 4 (Column 5)
6	-	0	0	17
5	6-5	7	7	10
4	6-4	5	5	12
3	6-5 , 5-3	7+6 = 13	13	4
2	6-5 , 5-2	7+1 = 8	8	9
1	6-5 , 5-2 , 2-1 6-5 , 5-3 , 3-1 6-4 , 4-1	7+1+2 = 10 7+6+4 = 17 5+3 = 8	17	0

Computation of Floats and slack

Activity (1)	Duration (2)	ES (3)	LF (4)	EF (5) = (2)+(3)	LS (6) = (4)-(2)	Total Float (7) = (6)-(3)	Head Event Slack (HES) (8)	Free Float (9) = (7)-(8)
1-2	2	0	9	2	7	7	7	0
1-3	4	0	4	4	0	0	0	0
1-4	3	0	12	3	9	9	9	0
2-5	1	2	10	3	9	7	0	7
3-5	6	4	10	10	4	0	0	0
4-6	5	3	17	8	12	9	0	9
5-6	7	10	17	17	10	0	0	0

Explanation

1. On the critical path 1-3-5-6, the total float or the slack is zero.
2. The duration of activity 1-2can be increased by 7 days or the duration of activity 2-5 can be increased by 7 days without affecting the total duration of the maintenance project (17 days).
3. The duration of activity 1-4 can be increased by 9 days or activity 4-6 can be increased by 9 days without affecting the total duration of the maintenance project (17 days).

Note :-

1. Total float is the amount of time an activity can be delayed without delaying the project completion date.
2. On a critical path, the total float is zero.
3. Total float is often known as the slack.
4. You can calculate the total float by subtracting the Early Start date of an activity from its Late Start date (Late Start date – Early Start date), or Early Finish date from its Late Finish date (Late Finish date – Early Finish date).
5. Free float is the amount of time an activity can be delayed without delaying the Early Start of its successor activity.
6. You can calculate the free float by subtracting the Early Finish date of the activity from the Early Start date of next activity (ES of next Activity – EF of current Activity).
7. If two activities are converging to a single activity, only one of these two activities may have free float.

Answer 6

This is the original cost matrix:

10	5	13	15	16
3	9	18	13	6
10	1	2	2	2
7	11	9	7	12
7	9	10	4	12

The solution will be obtained through Hungarian Method comprising of the following steps:-

Step I - Subtract row minima

We subtract the row minimum from each row:

5	0	8	10	11	(-5)
0	6	15	10	3	(-3)
9	0	1	1	1	(-1)
0	4	2	0	5	(-7)
3	5	6	0	8	(-4)

Step II - Subtract column minima

We subtract the column minimum from each column:

5	0	7	10	10
0	6	14	10	2
9	0	0	1	0
0	4	1	0	4
3	5	5	0	7
		(-1)		(-1)

Step III - Cover all zeros with a minimum number of lines

There are 4 lines required to cover all zeros:

5	**0**	**7**	**10**	**10**	**x**
0	6	14	**10**	2	
9	**0**	**0**	**1**	**0**	**x**
0	4	1	**0**	4	
3	5	5	**0**	7	
x			**x**		

Step IV - Create additional zeros

The number of lines is smaller than 5. The smallest uncovered number is 1. We subtract this number from all uncovered elements and add it to all elements that are covered twice:

6	0	7	11	10
0	5	13	10	1
10	0	0	2	0
0	3	0	0	3
3	4	4	0	6

Step V - Cover all zeros with a minimum number of lines

There are 5 lines required to cover all zeros:

6	**0**	**7**	**11**	**10**	**x**
0	**5**	**13**	**10**	**1**	**x**
10	**0**	**0**	**2**	**0**	**x**
0	**3**	**0**	**0**	**3**	**x**
3	**4**	**4**	**0**	**6**	**x**

Step V - The optimal assignment

Because there are 5 lines required, the zeros cover an optimal assignment:

6	**0**	7	11	10
0	5	13	10	1
10	0	0	2	**0**
0	3	**0**	0	3
3	4	4	**0**	6

This corresponds to the following optimal assignment in the original cost matrix:

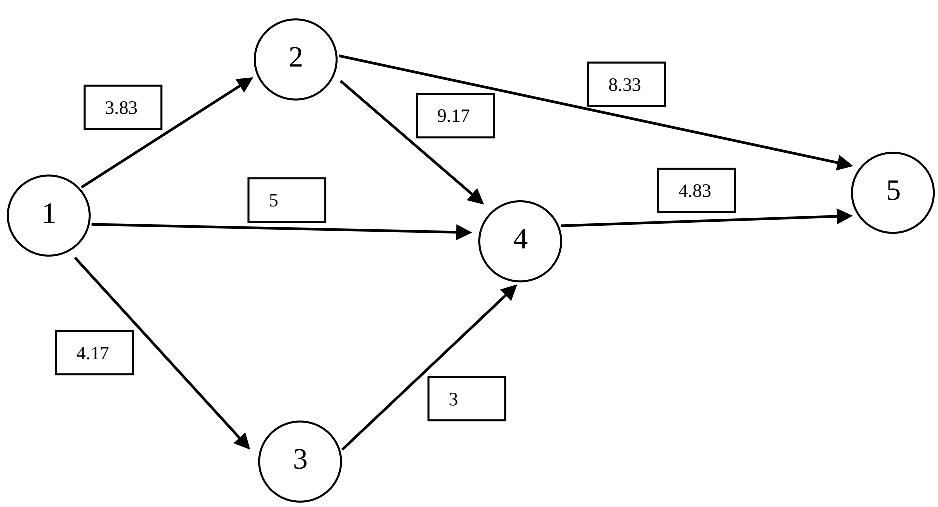
10	**5**	13	15	16
3	9	18	13	6
10	1	2	2	**2**
7	11	**9**	7	12
7	9	10	**4**	12

The optimal value equals 23 (5+3+2+9+4=23)

Answer 7

Activity	Time Estimates			Expected Time	Variance
	Optimistic (t_o)	Most Likely (t_m)	Pessimistic (t_p)	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$
1-2	2	4	5	3.83	1/4
1-3	3	4	6	4.17	1/4
1-4	4	5	6	5	1/9
2-4	8	9	11	9.17	1/4
2-5	6	8	12	8.33	1
3-4	2	3	4	3	1/9
4-5	2	5	7	4.83	25/36

Network Diagram



Critical Path

(i) Path 1 → 2 → 5 : 3.83+8.33 = 12.16

(ii) Path 1 → 2 → 4 → 5: 3.83+9.17+4.83 = 17.83
(iii) Path 1 → 3 → 4 → 5 : 4.17+3+4.83 = 12
(iv) Path 1 → 4 → 5 : 5+4.83 = 9.83

So, the critical path is 1 → 2 → 4 → 5 and the expected completion time is 17.83 days

Expected variance of completion time = sum total of variances of the critical path = (1/4)+(1/4)+(25/36) = 1.19